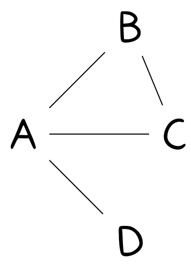
Part 1: Transition Probabilities

1. Consider the following undirected network. Represent the network with an adjacency matrix A.



2. A random walk traverses the network by randomly moving from one node to another. Fill the following table with the transition probabilities.

P	To A	To B	To C	To D
From A				
From B				
From C				
From D				

Part 2: Multiple Steps

3. If we start at node A, what is the probability of being at node C after exactly two steps? Show your calculation. Hint: First, calculate the probability $P(j|t=1)$ of being at each node j after one step. Then, multiply the probability of moving from each node j to node i, and sum up the probabilities. Namely,

$$P(i|t=2) = \sum_j \underbrace{P(i|j)}_{\substack{\text{Transition probability} \\ \text{from node } j \text{ to node } i}} \underbrace{P(j|t=1)}_{\substack{\text{Probability of being at} \\ \text{node } j \text{ after 1 step}}}$$

4. If we start at node A, what is the probability of being at node B after exactly three? Show your calculation.

5. How would you calculate the probability of being at any node after T steps, starting from node A? (You don't need to do the calculation, just describe the process using matrix multiplication.)

Part 3: Stationary Distribution

6. Calculate the probability distribution of being at each node after 1 step, 2 steps, ... 100 steps, starting from node A. Create a heatmap representation of the probability distributions for 1, 2, ... 100 steps, where rows represent nodes and columns represent steps. You can use a simple grid where darker shades represent higher probabilities. You may use pen and paper or computer software to create the heatmap.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
A																																												
B																																												
C																																												
D																																												
E																																												
F																																												

	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87
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7. Let's create another heatmap using a random walk starting from node D.

8. Based on your heatmap, what observations can you make about how the probabilities change as the number of steps increases?

9. What makes the stationary probability higher for some nodes than others?