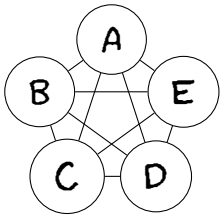
1 Perfect vs. Almost Perfect

You'll explore different types of friend groups in a class, focusing on various forms of pseudo-cliques. Compare the two given networks and answer the questions to discover concepts about pseudo-cliques.

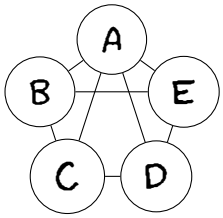
1.1 k-plex

Compare these two groups of 5 students:

GROUP A (PERFECT CLIQUE):



GROUP B (1-PLEX):



1. How many friends does each student have in Group A? _____
2. In Group B, how many students is each person not friends with?

   _____

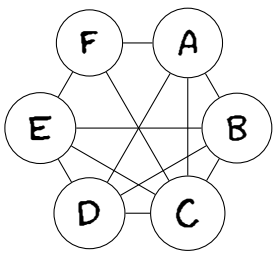
3. Which group represents a perfect clique? _____
4. Group B is called a 1-plex. Based on your observations, what do you think defines a k-plex?

   _____

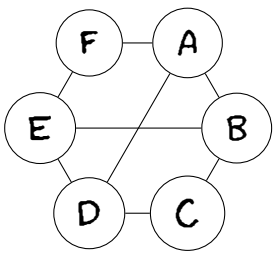
1.2 Core Friends (k-core)

Compare these two groups of 6 students:

GROUP A (3-CORE):



GROUP B (2-CORE):



5. What's the minimum number of friends any student has in Group A? _____

6. What's the minimum number of friends any student has in Group B? _____

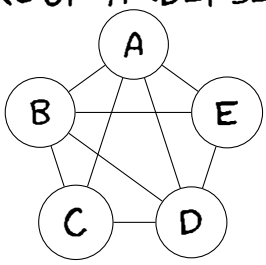
7. Group A is called a 3-core, and Group B is a 2-core. Based on your observations, what defines a k-core?

   _____

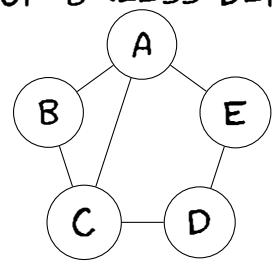
1.3 Density Matters (ρ-dense subgraphs)

Compare these two groups of 5 students:

GROUP A (DENSE):



GROUP B (LESS DENSE):



8. How many total friendships are there in Group A? _____

9. How many total friendships are there in Group B? _____

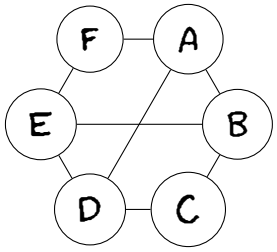
10. Calculate the density for each group using this formula: density = (number of edges) / (maximum possible edges)

    Group A density: _____ Group B density: _____

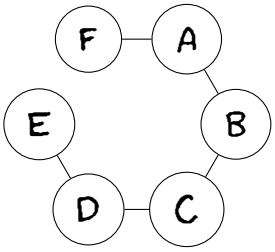
1.4 Extended Connections (n-clique)

Compare these two groups of 6 students:

GROUP A (2-CLIQUE):



GROUP B (NOT A 2-CLIQUE):



11. In Group A, what's the maximum distance from any student to any other? _____

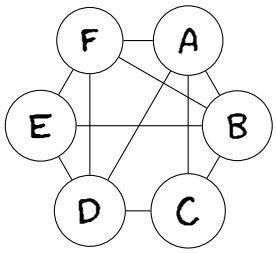
12. In Group B, what's the maximum distance from any student to any other? _____

13. Group A is called a 2-clique. Based on your observations, what do you think defines an n-clique?

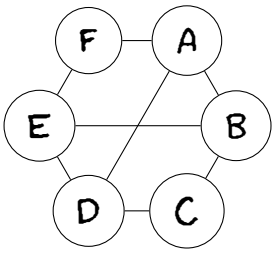
1.5 Triangles (k-truss)

Compare these two groups of 6 students:

GROUP A (3-TRUSS):



GROUP B (NOT A 3-TRUSS):



14. In Group A, how many triangles (groups of 3 mutually connected friends) does each edge participate in? _____

    (a) A-B: _____

    (b) B-C: _____

    (c) C-D: _____

    (d) D-E: _____

    (e) E-F: _____

    (f) F-A: _____

(g) A-C: _____
(h) A-D: _____
(i) B-E: _____
(j) B-F: _____
(k) D-F: _____

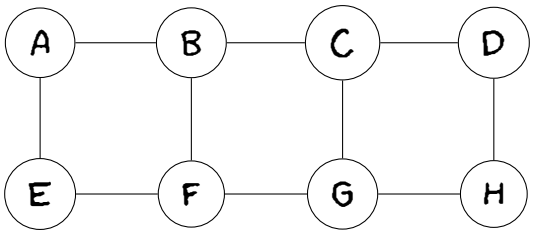
15. In Group B, do all edges participate in at least one triangle? _____
16. Group A is called a 3-truss. Based on your observations, what do you think defines a k-truss?
17. How might the k-truss be related to the k-core?

2 Partitioning

You will be given a network and asked to partition it into two communities. Answer the following questions for each network.

2.1 Graph Cut

Consider this network of 8 students:



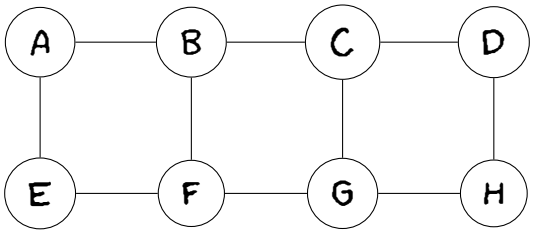
1. Draw a line to divide this network into two disconnected components. Explain why you chose to cut where you did. **Explanation:**

   _____

2. How many edges does your cut cross? _____
3. This number of crossed edges is called the "cut size". What might be a problem with always choosing the cut that gives the smallest cut size?

   _____

2.2 Balanced Cut



4. We want to balance the size of each community. The "size" of a community can be measured by the number of nodes it has, or the number of edges it has. The Ratio Cut does this by dividing the cut size by the number of nodes in each community:

$$\text{Ratio Cut} = \text{cut size} * \left(\frac{1}{|A|} + \frac{1}{|B|}\right),$$

where |A| and |B| are the number of nodes in each community. Find a cut that minimizes the Ratio Cut.

5. Normalized Cut instead uses the total degree (number of edges) of nodes in each community:

$$\text{Normalized Cut} = \text{cut size} * \left(\frac{1}{\text{vol}(A)} + \frac{1}{\text{vol}(B)}\right),$$

where vol(A) and vol(B) are the sum of the degrees of nodes in each community.

Calculate the Normalized Cut for the same division.

Normalized Cut = _____