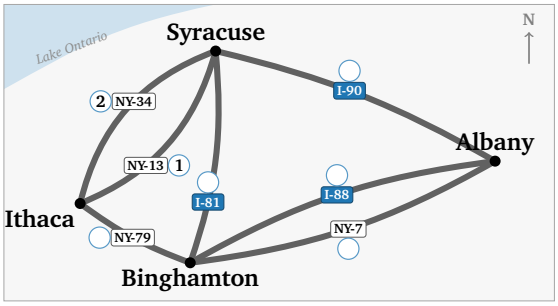
Part 1 — Seven highways

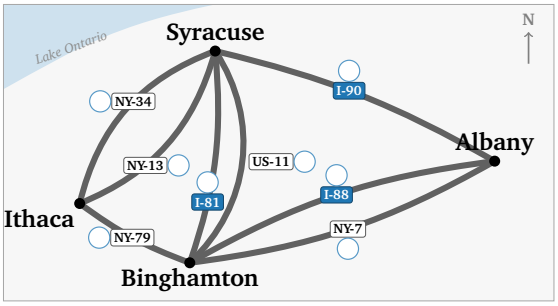
Seven highways join four Upstate New York cities: Ithaca, Syracuse, Binghamton, and Albany.

Question 1(a): Drive every highway exactly once, without lifting your pencil. Start and finish anywhere; cities may repeat, highways may not. Write in each circle when you drive that road — 1 first, then 2, on to 7. Two are filled in: you start at Ithaca, take NY-13, then NY-34 straight back.

Question 1(b): The same four cities, and an eighth highway: **US-11**, which really does run alongside I-81. Number your route again. Can you drive every highway without lifting your pencil?



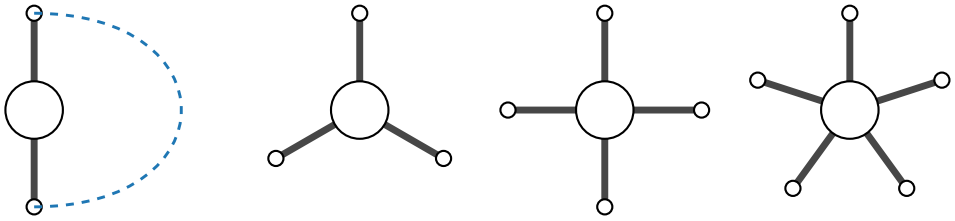
1(a): seven highways



1(b): with US-11

Question 2: One city, on its own. Arriving uses up one highway, leaving uses up another — so highways get used in **pairs**.

Join two road-ends with a curved line to mean "in here, out there". Pair up as far as you can. The first is done.



2 roads 3 roads 4 roads 5 roads

	2 roads	**3 roads**	**4 roads**	**5 roads**
pairs you drew	1			
roads left over	0			

(a) Which of these cities can you drive straight *through*?

(b) What do those numbers share?

(c) A left-over road has no partner. What must this city be for the tour?

Question 3: Count the highways at each city. Ithaca is done.

	I	**S**	**B**	**A**	**how many have a left-over?**
seven roads	3				
with US-11 too					

One start, one finish, so at most how many cities may have a left-over road?

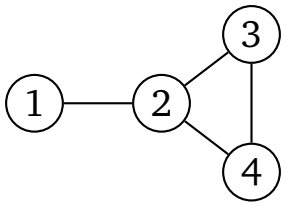
For the seven roads: can you drive every highway exactly once? If so, where does that drive begin and end?

For the eight roads, with US-11: can you drive every highway exactly once?

Why is there no route with eight?

Part 2 — Three kinds of route

A different network: four stops, numbered 1 to 4.



Question 4(a): Fill in the table. The last column takes one of these three names. First row done.

> Repeats nothing: **path**. Never repeats a line: **trail**. Repeats both: **walk**.

route	repeats a line?	repeats a stop?	name
1 2 3 2 4	yes (2–3 twice)	yes (2)	
1 2 3 4 2			
1 2 3 4			

Question 4(b): Write a route from 1 to 2 that repeats a stop but no line.

Question 4(c): Back to the highway map. Your drive in 1(a) used no road twice, but it did come back to a city it had already visited. Which is it — a path, a trail, or a walk?

Question 4(d): It could not have been a path. How many different cities would a path of 7 roads have to touch? How many does the highway map have? Why does that rule out a path?

Part 3 — Predictions

Part 4 hands you a notebook — a computer page that will run the network from Part 2 for you. Work the answers out on paper first; the notebook will check them.

Question 5(a): Write **1** in row i, column j if a line joins i and j, else **0**, for the network from Part 2. Row 1 done.

$A =$	1	2	3	4
1	0	1	0	0
2				
3				
4				

Question 5(b): Add up each row, then compare with the network from Part 2. What are the four row totals? What does each row total count, for that node?

Question 5(c): A **2-step route** goes along one line, then along one more. From node 1 to node 3, $1 \to 2 \to 3$ is one such route. Using the network from Part 2, count the 2-step routes:

How many go from 1 to 3? Write one out.

How many go from 1 to 2? Why so few?

How many go from node 2 back to node 2?

Question 5(d): The notebook in Part 4 will square your grid for you. Before it does, say what you expect to find in A^2.

What do you expect in row 1, column 3? In row 1, column 2?

What is on the diagonal of A^2, and why?

Part 4 — The lab, on your own

Do this one by yourself, with this sheet beside you. The notebook takes the same four cities and seven highways you have been drawing on, and asks you to write them down in a form a computer can read.

Point a camera at the square. If that fails, type `go.skojaku.com/m01lab`