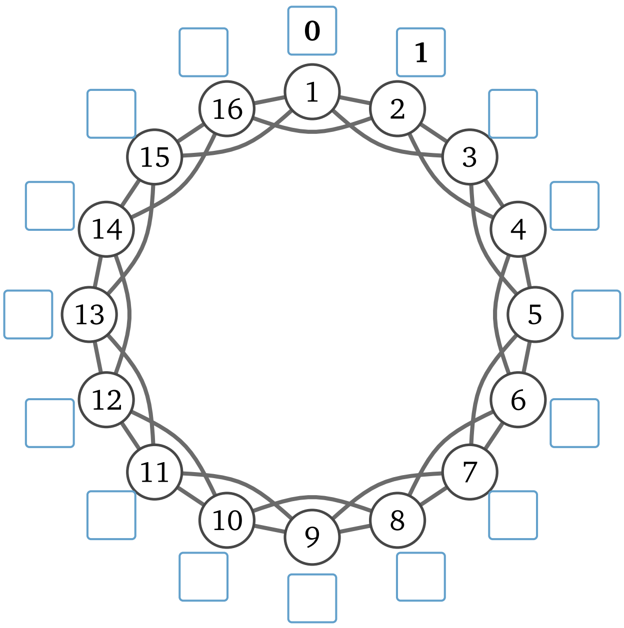
Part 1 — A letter from Omaha

In 1967 a man in Omaha, Nebraska was handed a letter addressed to a stockbroker outside Boston — a stranger, thirteen hundred miles away. He could not post it. He could pass it only to somebody he knew on first-name terms, who passed it on the same way.

Question 1: The letter arrived. How many hands did it pass through on the way? Write your guess, and write why you guessed that.

Part 2 — Ringville

Sixteen people live in Ringville and they sit in one circle. Everybody is friends with the two on their left and the two on their right: four friends each, thirty-two friendships in the town.



Question 2(a): Person 1 has a letter for person 9. Trace the chain on the drawing. How many handshakes does it take?

Question 2(b): Now everybody. In each person's box on the drawing, write how many handshakes it takes to reach them from person 1. Two are written in already, and you may find you only have to work out half of the rest.

Question 2(c): Add up every number in the boxes. What is the total?

Share that total out among the fifteen other people. That is the town's **average**.

And what is the biggest number in any box — the town's **worst case**?

Question 2(d): Take the average and the worst case. One of the two you would give to a newspaper; the other you would give to an engineer who has to promise the letter arrives. Which is which, and why?

Question 3: Ringville grows. A thousand people, still one circle, still four friends each.

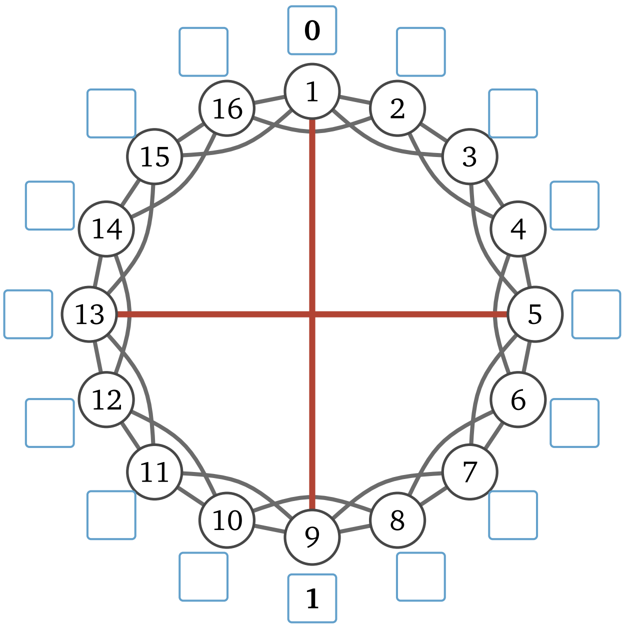
How many handshakes does it take to reach the person sitting directly opposite you?

And if the circle held all eight billion people on Earth?

Could a world wired like Ringville get the Omaha letter to Boston in six handshakes? Why?

Part 3 — Two shortcuts

Two of the sixteen went to college and came home with a friend from the far side of town: person 1 is now also friends with person 9, and person 5 with person 13 — thirty-four friendships, not thirty-two.



Question 4: Write the handshake counts into the boxes again, this time for the town with the two new friendships. Two are written in already.

Then work out the same three numbers as in 2(c): the new total, the new average, and the new worst case.

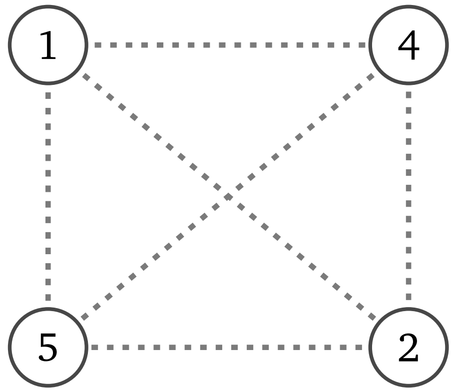
Question 5: Thirty-two friendships became thirty-four. Work out both changes as percentages: by how much did the number of friendships change, and by how much did the average change?

Say, in your own words, what those two new friendships did that thirty-two ordinary ones could not.

Part 4 — Do your friends know each other?

Go back to Ringville as it was in Part 2, before the two shortcuts, and take somebody ordinary. **Person 3** has four friends: persons 1, 2, 4 and 5.

Four people can be paired up six ways, and here are the six, lifted out of the circle as dashed lines.



person 3's four friends

Question 6(a): Check the six pairs against the circle in Part 2, one pair at a time. Darken the dashed lines that are real friendships. How many of the six are?

Question 6(b): Ringville grows to ten thousand people, still four friends each. How many of person 3's six pairs are friends now?

Question 6(c): A different town: the same sixteen people and the same thirty-two friendships, but this time drawn out of a hat.

Pick two people. What is the chance that they are friends?

So how many of person 3's six pairs would you expect to be friends?

And in a hat-drawn town of ten thousand, still four friends each?

Question 6(d): That fraction has a name — it is person 3's **local clustering**. You have now measured it four times. Say in one sentence what makes it big and what makes it small.

Three people who are all friends with each other make a **triangle**. Ringville, before the two shortcuts, has sixteen of them.

Question 7(a): Then the two shortcuts went in. Do persons 1 and 9 have a friend in common? Do persons 5 and 13? So how many triangles does the town have now?

Question 7(b): The real model does not *add* a shortcut, it **moves** one: person 1 drops person 2 and befriends person 9 instead.

Which friends did persons 1 and 2 have in common?

So how many triangles does the town lose?

Question 7(c): Put Question 5 beside 7(a) and 7(b). What does a shortcut buy the town, and what does it cost?

Part 5 — Back to Omaha

Question 8(a): Everybody knows 150 people. Pretend for a moment that no two of your friends know each other, so every handshake reaches 150 people nobody in the chain has met yet. Round as you like.

handshakes	people reached
1	150
2	22,500
3	about 3.4 million
4	
5	

Carry the table two rows further. At how many handshakes do you pass eight billion?

Compare that with your guess in Question 1.

Question 8(b): In Question 6 you found that person 3's friends do know each other, so the pretending in 8(a) is false. Which way does that push the answer — more handshakes, or fewer? Why?

Question 8(c): You are person 3 in the town with the two shortcuts, holding a letter for person 12. You cannot see the drawing: you know only your own four friends and the chair each of them sits in. Hand it to whichever of them sits closest to chair 12; they know only *their* friends and do the same. If two are equally close, pick either.

Play it out one person at a time and write the chain down. How many handshakes does it take?

Now look at the whole drawing in Part 3. What is the shortest chain from person 3 to person 12, and how long is it?

Question 8(d): Of 296 letters sent from Omaha, 64 arrived, in five or six steps. Is "a short chain exists" the same claim as "an ordinary person can find one"? Which of the two did the letters test, and what do the 232 that never arrived say about it?

Part 6 — The lab, on your own

Do this one by yourself, with this sheet beside you. The notebook builds the same town and counts its handshakes and its triangles for you — once you have written the three rules it runs on.

Point a camera at the square. If that fails, type `go.skojaku.com/m02lab`